

Statistics

Lecture 3



Feb 19-8:47 AM

$x \rightarrow$ Data element
 $\sum x \rightarrow$ Sum of data elements
 $n \rightarrow$ Sample Size

$\bar{x} \rightarrow$ x-bar $\rightarrow$ Sample Mean (Average)

$\bar{x} = \frac{\sum x}{n}$

Consider the Sample below
 2 4 4 4 8

$n = 5$ Range = $8 - 2 = 6$ Midrange = $\frac{8+2}{2} = 5$

Mode 4 Median 4
 the value in the middle
 (Data must be Sorted)

$\sum x = 2 + 4 + 4 + 4 + 8 = 22$

$\bar{x} = \frac{\sum x}{n} = \frac{22}{5} = 4.4$ Round to whole # $\rightarrow 4$
 Round up to whole # $\rightarrow 5$

SG 5

Sep 12-8:03 AM

$n \rightarrow$ Sample Size

$\bar{x} \rightarrow$ x -bar

$\rightarrow$ Sample Mean

$x \rightarrow$ Data element

$\sum x \rightarrow$ Sum of data elements $\bar{x} = \frac{\sum x}{n}$

$\sum x^2 \rightarrow$ Square every data element, then add them

$s^2 \rightarrow$ Sample Variance

$\rightarrow$ We use variance to find

standard deviation

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$s^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}$$

Variance is the Sum of square of the differences between every data element and the mean then divide by $n-1$.

Sep 12-8:11 AM

Consider the Sample below

1 4 4 6 6 9

$n = 6$

Range = $9 - 1 = 8$

Midrange = $\frac{9+1}{2} = 5$

Mode = 4 & 6

Median = $\frac{4+6}{2} = 5$

Value in the middle

(Data must be Sorted)

$$\sum x = 1 + 4 + 4 + 6 + 6 + 9 = 30$$

$$\sum x^2 = 1^2 + 4^2 + 4^2 + 6^2 + 6^2 + 9^2 = 186$$

$$\bar{x} = \frac{\sum x}{n} = \frac{30}{6} = 5$$

$$s^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}$$

$$s^2 \geq 0$$

$$= \frac{6 \cdot 186 - 30^2}{6(6-1)} = \frac{216}{30} = 7.2$$

It is never negative.

Sep 12-8:18 AM

$$\begin{aligned}\bar{x} &\rightarrow \text{Sample Mean} & \bar{x} &= \frac{\sum x}{n} \\ s^2 &\rightarrow \text{Sample Variance} & s^2 &= \frac{\sum (x - \bar{x})^2}{n-1} \\ & & s^2 &= \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}\end{aligned}$$

$s \rightarrow$ Sample Standard deviation

$$s = \sqrt{s^2} \quad \mathbf{s \geq 0}$$

Standard deviation is a value that indicates the deviation of data elements from the mean.

when s is small $\rightarrow$ data elements are close to $\bar{x}$.

when s is big $\rightarrow$ data elements are more spread out from the mean.

when s is zero $\rightarrow$ all data elements are the same and equal to $\bar{x}$.

Sep 12-8:26 AM

Consider the Sample below

1 2 2 2 3

$n=5$ Range = 2 Midrange = 2

Mode = 2 Median = 2

$$\sum x = 10$$

$$\sum x^2 = 22$$

$$\bar{x} = \frac{\sum x}{n} = \frac{10}{5} = \mathbf{2}$$

$$\begin{aligned}s^2 &= \frac{n \sum x^2 - (\sum x)^2}{n(n-1)} \\ &= \frac{5 \cdot 22 - 10^2}{5(5-1)} = \frac{10}{20} \\ &= \mathbf{.5}\end{aligned}$$

$$s = \sqrt{s^2} = \sqrt{.5} \approx \mathbf{.707}$$

Small $\uparrow$

there is not that much deviation between $\bar{x}=2$ and data elements
1 2 2 2 3

Sep 12-8:35 AM

Consider the Sample below

1 2 2 2 30

$$\sum x = 37$$

$$\sum x^2 = 913$$

$$\bar{x} = \frac{\sum x}{n} = \frac{37}{5} = 7.4$$

$$S^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}$$

$$= \frac{5 \cdot 913 - 37^2}{5(5-1)} = \frac{3196}{20} = 159.8$$

$$S = \sqrt{S^2} = \sqrt{159.8}$$

$$\text{big } \approx 12.641$$

Since S is a big #,
there is more
deviation between
data elements and
the Sample mean $\bar{x}$.

Sep 12-8:41 AM

Given $n = 10$ $\sum x = 80$ $\sum x^2 = 640$

Find

$$\bar{x} = \frac{\sum x}{n} = \frac{80}{10} = 8$$

$$S^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)} = \frac{10 \cdot 640 - 80^2}{10(10-1)}$$

$$S = \sqrt{S^2} = \sqrt{0} = 0$$

$$= \frac{0}{90} = 0$$

what do you conclude about data elements? Since $S = 0$, all data elements are the same and equal to $\bar{x} = 8$

How to estimate Sample Standard deviation:

$$S \approx \frac{\text{Range}}{4}$$

The range rule-of-thumb.

Sep 12-8:48 AM

Given $n=20$ $\sum x = 1583$
 $\sum x^2 = 128743$ $\text{Min} = 52$ $\text{Max} = 100$

Find

$$\bar{x} = \frac{\sum x}{n} = \frac{1583}{20} = \boxed{79.15} \quad s^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)} = \frac{20 \cdot 128743 - 1583^2}{20(20-1)}$$

$$s = \sqrt{s^2} = \sqrt{181.503} = \frac{68971}{380} \approx \boxed{181.503}$$

$$\approx \boxed{13.472} \approx \boxed{13}$$

Estimate s

$$\approx \frac{\text{Range}}{4} = \frac{100-52}{4} = \frac{48}{4} = \boxed{12}$$

Sep 12-8:57 AM

When mean, mode, and median are the same, data dist. will be symmetric and bell-shape looking.

Empirical Rule 68% - 95% - 99.7% Ranges

About 68% of data elements are within $\bar{x} \pm s$

About 95% of data elements are within $\bar{x} \pm 2s$
 Usual Range

About 99.7% of data elements are within $\bar{x} \pm 3s$

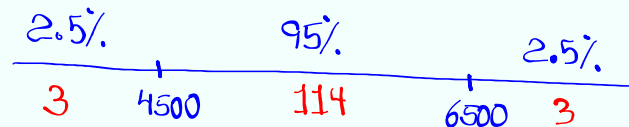
Sep 12-9:08 AM

I randomly Selected 120 nurses, their Salaries had a bell-Shape dist with $\bar{x} = \$5500/\text{mo.}$ with $S = \$500/\text{mo.}$

68% Range $\rightarrow \bar{x} \pm S = 5500 \pm 500$

68% of 120 ≈ 82 of them $\Rightarrow$ 5000 to 6000

95% Range $\rightarrow \bar{x} \pm 2S = 5500 \pm 2(500)$
 $\Rightarrow$ 4500 to 6500



95% of 120 = 114

Sep 12-9:13 AM

Z-Score Round to 3-dec. places

$$Z = \frac{x - \bar{x}}{S}$$

Z-Scores indicates how many Standard deviation is the data element from $\bar{x}$.

$-2 \leq Z \leq 2$ $\leftrightarrow$ usual data element

$Z < -2$ or $Z > 2$ $\leftrightarrow$ Unusual data element

Z-Score allows us to compare data elements from different Samples

It is process of Standardize data elements.

Sep 12-9:19 AM

Lisa makes \$4000 as a nurse while

Mike makes \$6000 as a Store manager.

who is performing better?

Nurse $\rightarrow \bar{x} = 3500$, $S = 400$

Managers $\rightarrow \bar{x} = 5000$, $S = 1000$

Lisa $Z = \frac{x - \bar{x}}{S} = \frac{4000 - 3500}{400} = 1.25$ usual

Mike $Z = \frac{x - \bar{x}}{S} = \frac{6000 - 5000}{1000} = 1$ usual

I have a Z-Score of -2.25 as a Store manager, what is my salary?

$$Z = \frac{x - \bar{x}}{S} \quad -2.25 = \frac{x - 5000}{1000}$$

Cross-Multiply $-2.25(1000) = x - 5000$

Solve for x $-2.25(1000) + 5000 = x$

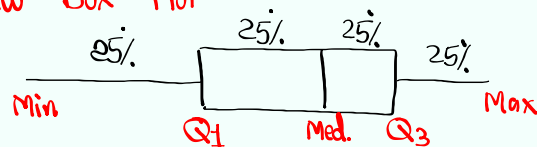
$$\boxed{x = 2750}$$

Sep 12-9:23 AM

5- Number Summary

Min Q_1 Med. Q_3 Max

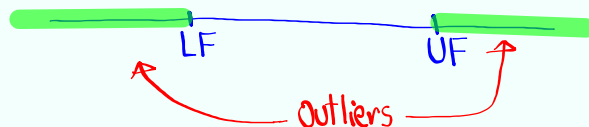
Draw Box Plot



$$IQR \text{ (Inter-Quartile-Range)} = Q_3 - Q_1$$

$$\text{upper Fence} = Q_3 + 1.5(IQR)$$

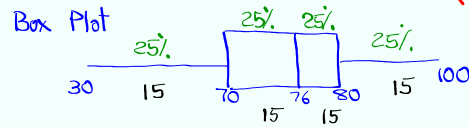
$$\text{Lower Fence} = Q_1 - 1.5(IQR)$$



Sep 12-9:45 AM

I randomly Selected 60 exams and here are the 5-Number Summary of Scores

30 70 76 80 100
 min Q_1 med. Q_3 Max } $60 \div 4 = 15$



25% below Q_1 , 75% above Q_1

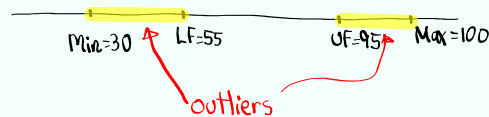
50% = Med., 50% above Med.

75% below Q_3 , 25% above Q_3

$$IQR = Q_3 - Q_1 = 80 - 70 = 10$$

$$\text{Upper Fence} = Q_3 + 1.5(IQR) = 80 + 1.5(10) = 95$$

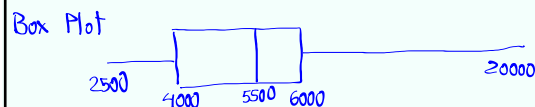
$$\text{Lower Fence} = Q_1 - 1.5(IQR) = 70 - 1.5(10) = 55$$



Sep 12-9:50 AM

5-Number Summary of monthly Salaries of nurses randomly Selected were

2500 4000 5500 6000 20000



$$IQR = Q_3 - Q_1 = 2000$$

$$\text{Upper Fence} = Q_3 + 1.5(IQR) = 6000 + 1.5(2000) = 9000$$

$$\text{Lower Fence} = Q_1 - 1.5(IQR) = 4000 - 1.5(2000) = 1000$$

outliers → 9000 to 20000



SG 5

You can work on SG 6.

Look for my email.

Sep 12-9:59 AM

TI Instructions:

1) To clear the Screen

Clear

2) To quit

2nd **MODE**

3) To clear all lists.

2nd **+** **4:clearAll lists** **Enter**

4) To reset all lists.

STAT **Edit** **Enter**
5:Set Up Editor

Sep 12-10:09 AM

How to store a data Set in a list:

Consider the Sample below

8 12 5 20 15

10 5 14 15 10

STAT **Edit**
1:Edit

L1
8 enter
12 enter
:
10 enter

Let's quit & clear Screen

2nd **Mode** **Clear**

Sep 12-10:15 AM

How to view a list.

I want to see what is in **L1**.

2nd **1** **Enter**

{ 8 12 5 20 ... ▶ }

go right → → →

go left ← ← ←

How to Sort a list:

STAT **Edit** **2nd** **1** **Enter**
 2:SortA(**L1**

Let's quit & clear Screen

end **Mode** **clear**

Sep 12-10:19 AM

Let's view **L1**:

2nd **1** **Enter**

{ 5 5 8 10 ... ▶ }

→ → → Right

Left ← ← ←

Sep 12-10:25 AM

How to find $\bar{x}$ & S using TI:

STAT $\rightarrow$ **CALC**
1: 1-Var Stats **2nd** **1**

No Menu

L1 **Enter**

With Menu

List: **L1**

Freq List: **Clear**

Calculate

Min = 5

$Q_1 = 8$

Med = 11

$Q_3 = 15$

Max = 20

5-Number
Summary

$\bar{x} = 11.4$

$S = S_x = 4.766$

$\downarrow n = 10$

$\downarrow$

$\downarrow$

Sep 12-10:27 AM

what about S^2 ?

VARs **5: Statistics** **3: S_x** **x^2** **Enter**

$S^2 = 22.71$

write this in reduced fraction

MATH **1: $\rightarrow$ Frac** **Enter** $\frac{1022}{45}$

Let's clear all lists

2nd **+** **4: Clear All lists** **Enter**

Sep 12-10:33 AM

I randomly Selected 20 students, here are their ages:

23	27	18	20	30
35	32	25	19	40
48	50	45	38	19
24	25	50	46	44

quit & clear Screen

2nd **Mode** **clear**

1) Store in L1.

STAT **Edit**
1: Edit

L1	
23	enter
27	"
18	"
:	"
44	"

Sep 12-10:40 AM

Sort L1, view L1, then make STEM plot

STAT **Edit** **2nd** **1** **Enter**
2: SortA

2nd **1** **Enter**

{ 18 19 19 20

→ → →

1		8 9 9
2		0 3 4 5 5 7
3		0 2 5 8
4		0 4 5 6 8
5		0 0

Sep 12-10:45 AM

Find $\bar{x}$ & S

STAT → **CALC**

1: 1-Var Stats

with Menu
List: L1
Freq list: **clear**
(Calculate)

end **|** **1**

No Menu
L1 **|** **Enter**

$\bar{x} = 32.9$
 $S = S_x = 11.369$

Min = 18
 $Q_1 = 23.5$
Med = 31
 $Q_3 = 44.5$
Max = 50

Find S^2 in reduced fraction

VARs
5: Statistics
3: Sx
 x^2 **Math** **1: $\rightarrow$ Svar** **Enter**

$S^2 = \frac{12279}{95}$

$n = 20$

Look for my email

Look for a couple of online Q&A.

Sep 12-10:50 AM